Representation the All Order of Every Element of 60 Order of Group for Multiplication Composition

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Abstract: This paper aims at treating a study on the order of every element of 60 orders of group for multiplication composition. But the composition in G is associative; the multiplication composition is very significant in the order of elements of a group. We develop the order of a group, higher order of groups in different types of order and the order of elements of a group in real numbers. Let G be a group and let $a^n \in G$ be of infinite order n. In addition, notation $o(a^m) = \frac{\lambda}{m}$, $where\ \lambda = l.c.m\ of\ m\ and\ n$. If $a \in G$ is of order n, then there exists an integer m for which $a^m = e$ if m is a multiple of n, in general we use this. Then we develop orders of elements of a cyclic group and every element of higher order of a group. After that we find out the order of every element of a group for the higher orders of the group for being binary operation.

Keywords: *$o(G)$, $o(a)$, Multiplication Composition, LCM, Torsion Group.*

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I. INTRODUCTION

We propose to study the groups of order of an element of a group, order of a group, torsion group, mixed group subgroup, normal subgroup and the integral powers of an element of a group etc. Then we discuss the order of every element in the higher 60 orders of group for multiplication composition. The group notation is o or *. We will frequently omit the symbol for the group operation but we will also often write the operation as · or + when it represents multiplication or addition in a group, and write 1or 0 for the corresponding identity elements respectively. It's addition +, multiplication × or (.) is used as a binary operation. If the group operation is denoted as a multiplication, then an element $a \in G$ is said to be order n if n is the least positive integer such that $a^n = e$ or $O(a) \leq n$ i. e., if $a^n = e\ and\ a^r \neq e\ \ \forall\ r \in N\ s.t.\ r < n$ .The order of a is denoted by $O(a)$. If$a^n \neq e\ \ for\ any\ n \in N$, then a is said to be of zero order or infinite order [1]. Let e is the identity element in (G, +). An element $a \in G$ is said to be order n if $n \in Z^+$ such that $na = e$ or $O(a) \leq n$. i. e., if $na = e\ and\ ar \neq e\ \ \forall\ r \in N\ s.t. 0 < r < n$ .The order of a is denoted by $O(a)$. If $na \neq e\ \ for\ any\ n \in N$, then a is said to be of zero order or infinite order [2]. The order of a group G and the orders of its elements give much information about the structure of the group. The order of any subgroup of G divides the order of G. If H is a subgroup of G, then $ord(G)\ /\ ord(H) = [G : H]$, where $[G : H]$ is called the index of H in G. This is Lagrange's theorem; however, it is only true when G has finite order. If $ord(G) = \infty$, the quotient $ord(G)\ /\ ord(H)$ is not true. We see that the order of every element of a group divides the order of the group. For example, in the symmetric group shown above, where $ord(S_3) = 6$, the possible orders of the elements a, b, c. But there is no general formula relating the order of a product ab to the orders of a and b. In fact, it is possible that both a and b have finite order while ab has infinite order, or that both a and b have infinite order while ab has finite order. (i.e. [3], [4], [5]). Then we find out the order of every element of a group in different types of the higher even, odd and prime order of the group for composition [6], [7].

II. INTEGRAL POWERS OF AN ELEMENT OF A GROUP

➢ *Multiplication Composition [8], [9]*

Let (G, .) be a group. Let $a \in G$ be an arbitrary element.

By closure property, all the elements a, aa, aaa, . . .etc. belong to G.

Since the composition in G is associative. Hence aaa . . . to n factors is independent of the manner in which the factors are grouped.

If n is a positive integer, then define

$a^n = a.a.a\ \ .\ .\ .\ to\ n\ factors$

$a^n \in G$, by closure property

If e is identity in G, then we define $a^0 = e$.

If n is a negative integer, then by define $a^{-n} = (a^n)^{-1}$, where $(a^n)^{-1}$ is the inverse of a^n

Consequently, $(a^n)^{-1} \in G$, since the inverse of every element of G belong to G. $\therefore\ a^{-n} \in G$

According to the definition

$$(a^n)^{-1} = (aaa\ .\ .\ .\ \text{to n factors})^{-1}$$
$$= (a^{-1})(a^{-1})(a^{-1})\ .\ .\ .\ \text{to n factors}$$
$$= (a^{-1})^n$$
$$\therefore\ a^{-n} = (a^n)^{-1} = (a^{-1})^n.$$

The following law of indices can be easily proved

$$(a^m)^n = a^{mn} \quad \forall\ a \in G \text{ and } \forall\ m, n \in Z$$
$$\text{and } a^m a^n = a^{m+n} \quad \forall\ a \in G \text{ and } \forall\ m, n \in Z$$

Thus we defined a^n for all integral values of n, positive, negative or zero.

III. SIGNIFICANCE OF THE ORDER OF AN ELEMENT OF A GROUP

We begin this section of the following theorem related significance of the order of an element of a group.

➢ *Theorem [10]:*

Let G be a group and let a ∈ G be of infinite order n. Then show that O $(a)^k = \frac{n}{(n,k)}$ where k is any integer and (n, k) is denoted the highest common factor of n and k.

Proof: Let o(a) = n ,$o(a)^k$ = h, (n, k) = m.

Now we will prove that $h = \frac{n}{m}$.

We have,

o(a) = n (1)

or, a^n = e

or, $o(a)^k$ =h

or, $(a^k)^h$ = e (2)

or, a^{kh} = e

And, (n, k) = m

or, n = mp , k = mg ; where p and q integers and (p, q) = 1

From (2) we get,

a^{kh} = e

Or, $a^{kh} = a^n$ or, kh = n, or, (kh) or, (mqh) or, (qh) or, (h) where

(p, q) = 1 (3)

Now, $a^{kp} = a^{(mq)p}$

= $a^{(mp)q}$[By Associative law]

= a^{nq}

= $(a^n)^q$

= e^q

or, a^{kp} = e (4)

or, $(a^k)^p$ = e

or, o(k) = p

or, h = p (5)

or, h / p

From (3) and (5) then we get,

P = h

or, h = $\frac{n}{m}$**QED**

➢ *Theorem [11]:*

Show that the order of every element of a finite group is finite.

- Proof: Let G be a finite group with multiplication composition.

Let $a \in G$ be an arbitrary element.

Now we will prove that $O(a)$ is finite.

By closure property, all the elements a^2=a. a, a^3=a. a. a, . . . etc. belong to G

i.e. a, a^2, a^3, a^4, a^5, a^6, a^7, . . . etc. belong to G.

But all these elements are not distinct. Since G is finite.

Let e be the identity in G, then $a^0 = e$.

Let us suppose that

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$a^m = a^n$ where $m > n$.

$\Rightarrow a^m a^{-n} = a^n a^{-n} = a^0 = e$

$\Rightarrow a^{m-n} = e \Rightarrow a^p = e$, where $p = m - n > 0$, as $m > n$

Also m and n are finite and hence p is a finite positive integer.

Now p is a positive integer s.t. $a^p = e$.

This proves that

$o(a) \le p =$ finite number

i.e. $o(a) \le$ a finite number $\Rightarrow o(a)$ is finite

IV. RESULT AND DISCUSSION

In this section we developed the result of order of every element for multiplication composition in the higher 60 orders of group for multiplication composition.

➢ *([12],[13], [14]) Find the Order of Every Element in the Multiplication Group*

$$\boldsymbol{G = \{a, \quad a^2, \quad a^3, \quad a^4, \ldots, a^{60} = e\}}$$

- *Solution:*
  The identity element of the given group is $a^{60} = e \Rightarrow o(a) = 60 \;\therefore\; o(a) = 60$

We know that $o(a^m) = \frac{\lambda}{m}$, $where\ \lambda = l.c.m\ of\ m\ and\ n$

To determine $o(a^2)$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 2\ is\ 60 \;\therefore o(a^2) = \frac{60}{2} = 30 \Rightarrow o(a^2) = 30$

To determine $o(a^3)$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 3\ is\ 60 \;\therefore o(a^3) = \frac{60}{3} = 20 \Rightarrow o(a^3) = 20$

To determine $o(a^4)$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 4\ is\ 60 \;\therefore o(a^4) = \frac{60}{4} = 15 \Rightarrow o(a^4) = 15$

To determine $o(a^5)$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 5\ is\ 60 \;\therefore o(a^5) = \frac{60}{5} = 12 \Rightarrow o(a^5) = 12$

To determine $o(a^6)$

Here, $o(a^{50}) = e, l.c.m\ of\ 60\ and\ 6\ is\ 60 \;\therefore o(a^6) = \frac{60}{6} = 10 \Rightarrow o(a^6) = 10$

To determine $o(a^7)$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 7\ is\ 420 \;\therefore o(a^7) = \frac{420}{7} = 60 \Rightarrow o(a^7) = 60$

To determine $o(a^8)$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 8\ is\ 120 \;\therefore o(a^8) = \frac{120}{8} = 15 \Rightarrow o(a^8) = 15$

To determine $o(a^9)$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 9\ is\ 180 \;\therefore o(a^9) = \frac{180}{9} = 20 \Rightarrow o(a^9) = 20$

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To determine $o(a^{10})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 10\ is\ 60\ \ \therefore o(a^{10}) = \frac{60}{10} = 6 \Rightarrow o(a^{10}) = 6$

To determine $o(a^{11})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 11\ is\ 660\ \ \therefore o(a^{11}) = \frac{660}{11} = 60 \Rightarrow o(a^{11}) = 60$

To determine $o(a^{12})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 12\ is\ 60\ \ \therefore o(a^{12}) = \frac{60}{12} = 5 \Rightarrow o(a^{12}) = 5$

To determine $o(a^{13})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 13\ is\ 780\ \ \therefore o(a^{13}) = \frac{780}{13} = 60 \Rightarrow o(a^{13}) = 60$

To determine $o(a^{14})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 14\ is\ 420\ \ \therefore o(a^{14}) = \frac{420}{14} = 30 \Rightarrow o(a^{14}) = 30$

To determine $o(a^{15})$

Here, $o(a^{60}) = e,\ l.c.m\ of\ 60\ and\ 15\ is\ 150\ \ \therefore o(a^{15}) = \frac{60}{15} = 4 \Rightarrow o(a^{15}) = 4$

To determine $o(a^{16})$

Here, $o(a^{60}) = e,\ l.c.m\ of\ 60\ and\ 16\ is\ 240\ \ \therefore o(a^{16}) = \frac{240}{16} = 15 \Rightarrow o(a^{16}) = 15$

To determine $o(a^{17})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 17\ is\ 1020\ \ \therefore o(a^{17}) = \frac{1020}{17} = 60 \Rightarrow o(a^{17}) = 60$

To determine $o(a^{18})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 18\ is\ 180\ \ \therefore o(a^{18}) = \frac{180}{18} = 10 \Rightarrow o(a^{18}) = 10$

To determine $o(a^{19})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 19\ is\ 1140\ \ \therefore o(a^{19}) = \frac{1140}{19} = 60 \Rightarrow o(a^{19}) = 60$

To determine $o(a^{20})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 20\ is\ 20\ \ \therefore o(a^{20}) = \frac{60}{20} = 3 \Rightarrow o(a^{20}) = 3$

To determine $o(a^{21})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 21\ is\ 420\ \ \therefore o(a^{21}) = \frac{420}{21} = 20 \Rightarrow o(a^{21}) = 20$

To determine $o(a^{22})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 22\ is\ 660\ \ \therefore o(a^{22}) = \frac{660}{22} = 30 \Rightarrow o(a^{22}) = 30$

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To determine $o(a^{23})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 23\ is\ 1380\ \ \therefore o(a^{23}) = \frac{1380}{23} = 60 \Rightarrow o(a^{23}) = 60$

To determine $o(a^{24})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 24\ is\ 120\ \ \therefore o(a^{24}) = \frac{120}{24} = 5 \Rightarrow o(a^{24}) = 5$

To determine $o(a^{25})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 25\ is\ 300\ \ \therefore o(a^{25}) = \frac{300}{25} = 12 \Rightarrow o(a^{25}) = 12$

To determine $o(a^{26})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 26\ is\ 780\ \ \therefore o(a^{26}) = \frac{780}{26} = 30 \Rightarrow o(a^{26}) = 30$

To determine $o(a^{27})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 27\ is\ 540\ \ \therefore o(a^{27}) = \frac{540}{27} = 20 \Rightarrow o(a^{27}) = 20$

To determine $o(a^{28})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 28\ is\ 420\ \ \therefore o(a^{28}) = \frac{420}{28} = 15 \Rightarrow o(a^{28}) = 15$

To determine $o(a^{29})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 29\ is\ 1740\ \ \therefore o(a^{29}) = \frac{1740}{29} = 60 \Rightarrow o(a^{29}) = 60$

To determine $o(a^{30})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 30\ is\ 60\ \ \therefore o(a^{30}) = \frac{60}{30} = 2 \Rightarrow o(a^{25}) = 2$

To determine $o(a^{31})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 31\ is\ 1860\ \ \therefore o(a^{31}) = \frac{1860}{31} = 60 \Rightarrow o(a^{31}) = 60$

To determine $o(a^{32})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 32\ is\ 480\ \ \therefore o(a^{32}) = \frac{480}{32} = 15 \Rightarrow o(a^{32}) = 15$

To determine $o(a^{33})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 33\ is\ 660\ \ \therefore o(a^{33}) = \frac{660}{33} = 20 \Rightarrow o(a^{33}) = 20$

To determine $o(a^{34})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 34\ is\ 1020\ \ \therefore o(a^{34}) = \frac{1020}{34} = 30 \Rightarrow o(a^{34}) = 30$

To determine $o(a^{35})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 35\ is\ 420\ \ \therefore o(a^{35}) = \frac{420}{35} = 12 \Rightarrow o(a^{35}) = 12$

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To determine $o(a^{36})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 36\ is\ 180\ \ \therefore o(a^{36}) = \frac{180}{36} = 5 \Rightarrow o(a^{36}) = 5$

To determine $o(a^{37})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 37\ is\ 2220\ \ \therefore o(a^{37}) = \frac{2220}{37} = 60 \Rightarrow o(a^{37}) = 60$

To determine $o(a^{38})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 38\ is\ 1140\ \ \therefore o(a^{38}) = \frac{1140}{38} = 30 \Rightarrow o(a^{38}) = 30$

To determine $o(a^{39})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 39\ is\ 780\ \ \therefore o(a^{39}) = \frac{780}{39} = 20 \Rightarrow o(a^{39}) = 20$

To determine $o(a^{40})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 40\ is\ 120\ \ \therefore o(a^{40}) = \frac{120}{40} = 3 \Rightarrow o(a^{40}) = 3$

To determine $o(a^{41})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 41\ is\ 2460\ \ \therefore o(a^{41}) = \frac{2460}{41} = 60 \Rightarrow o(a^{41}) = 60$

To determine $o(a^{42})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 42\ is\ 420\ \ \therefore o(a^{42}) = \frac{420}{42} = 10 \Rightarrow o(a^{42}) = 10$

To determine $o(a^{43})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 43\ is\ 2580\ \ \therefore o(a^{43}) = \frac{2580}{43} = 60 \Rightarrow o(a^{43}) = 60$

To determine $o(a^{44})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 44\ is\ 660\ \ \therefore o(a^{44}) = \frac{660}{44} = 15 \Rightarrow o(a^{44}) = 15$

To determine $o(a^{45})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 45\ is\ 180\ \ \therefore o(a^{45}) = \frac{180}{45} = 4 \Rightarrow o(a^{45}) = 4$

To determine $o(a^{46})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 46\ is\ 1380\ \ \therefore o(a^{46}) = \frac{1380}{46} = 30 \Rightarrow o(a^{46}) = 30$

To determine $o(a^{47})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 47\ is\ 2820\ \ \therefore o(a^{47}) = \frac{2820}{47} = 60 \Rightarrow o(a^{47}) = 60$

To determine $o(a^{48})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 48\ is\ 240\ \ \therefore o(a^{48}) = \frac{240}{48} = 5 \Rightarrow o(a^{48}) = 5$

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To determine $o(a^{49})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 49\ is\ 2940\ \ \therefore o(a^{49}) = \frac{2940}{49} = 60 \Rightarrow o(a^{49}) = 60$

To determine $o(a^{50})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 50\ is\ 300\ \ \therefore o(a^{50}) = \frac{300}{50} = 6 \Rightarrow o(a^{50}) = 6$

To determine $o(a^{51})$

Here, $o(a^{51}) = e, l.c.m\ of\ 60\ and\ 51\ is\ 1020\ \ \therefore o(a^{46}) = \frac{1020}{51} = 20 \Rightarrow o(a^{51}) = 20$

To determine $o(a^{52})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 52\ is\ 780\ \ \therefore o(a^{47}) = \frac{780}{52} = 15 \Rightarrow o(a^{52}) = 15$

To determine $o(a^{53})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 53\ is\ 3180\ \ \therefore o(a^{53}) = \frac{3180}{53} = 60 \Rightarrow o(a^{53}) = 60$

To determine $o(a^{54})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 54\ is\ 540\ \ \therefore o(a^{54}) = \frac{540}{54} = 10 \Rightarrow o(a^{54}) = 10$

To determine $o(a^{55})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 55\ is\ 660\ \ \therefore o(a^{55}) = \frac{660}{55} = 12 \Rightarrow o(a^{55}) = 12$

To determine $o(a^{56})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 56\ is\ 840\ \ \therefore o(a^{56}) = \frac{840}{56} = 15 \Rightarrow o(a^{56}) = 15$

To determine $o(a^{57})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 57\ is\ 1140\ \ \therefore o(a^{57}) = \frac{1140}{53} = 20 \Rightarrow o(a^{57}) = 20$

To determine $o(a^{58})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 58\ is\ 1740\ \ \therefore o(a^{58}) = \frac{1740}{58} = 30 \Rightarrow o(a^{58}) = 30$

To determine $o(a^{59})$

Here, $o(a^{60}) = e, l.c.m\ of\ 60\ and\ 59\ is\ 3540\ \ \therefore o(a^{59}) = \frac{3540}{59} = 60 \Rightarrow o(a^{59}) = 60$

Hence $\boldsymbol{G} = \{\mathbf{7},\ \mathbf{7^2},\ \mathbf{7^3},\ \mathbf{7^4},\ \ldots,\mathbf{7^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{11},\ \mathbf{11^2},\ \mathbf{11^3},\ \mathbf{11^4},\ \ldots,\mathbf{11^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{13},\ \mathbf{13^2},\ \mathbf{13^3},\ \mathbf{13^4},\ \ldots,\mathbf{13^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{17},\ \mathbf{17^2},\ \mathbf{17^3},\ \mathbf{17^4},\ \ldots,\mathbf{17^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{19},\ \mathbf{19^2},\ \mathbf{19^3},\ \mathbf{19^4},\ \ldots,\mathbf{19^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{23},\ \mathbf{23^2},\ \mathbf{23^3},\ \mathbf{23^4},\ \ldots,\mathbf{23^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{29},\ \mathbf{29^2},\ \mathbf{29^3},\ \mathbf{29^4},\ \ldots,\mathbf{29^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{31},\ \mathbf{31^2},\ \mathbf{31^3},\ \mathbf{31^4},\ \ldots,\mathbf{31^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{37},\ \mathbf{37^2},\ \mathbf{37^3},\ \mathbf{37^4},\ \ldots,\mathbf{37^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{41},\ \mathbf{41^2},\ \mathbf{41^3},\ \mathbf{41^4},\ \ldots,\mathbf{41^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{43},\ \mathbf{43^2},\ \mathbf{43^3},\ \mathbf{43^4},\ \ldots,\mathbf{43^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{47},\ \mathbf{47^2},\ \mathbf{47^3},\ \mathbf{47^4},\ \ldots,\mathbf{47^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{49},\ \mathbf{49^2},\ \mathbf{49^3},\ \mathbf{49^4},\ \ldots,\mathbf{49^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{53},\ \mathbf{53^2},\ \mathbf{53^3},\ \mathbf{53^4},\ \ldots,\mathbf{53^{60}} = \boldsymbol{e}\}$, $\boldsymbol{G} = \{\mathbf{59},\ \mathbf{59^2},\ \mathbf{59^3},\ \mathbf{59^4},\ \ldots,\mathbf{59^{60}} = \boldsymbol{e}\}$

V. ANALYSIS

We discussed the result of order of every element for multiplication composition in the 60 orders of group for multiplication composition. In fact, we can use composition related theorem to evaluate order of group of different orders such as order $2, 3, 4, 5, \ldots, 20$ etc., i.e., whose order is not so high (Not Higher Order Groups). As a result, we use multiplication related theorems to evaluate the order of groups of the higher order of group for composition. Here, to find orders of elements of a cyclic group. Thus, it has been found necessary and convenient to work or solve these structures in details.

VI. CONCLUSION

In this work we developed the higher order of elements of a group for various higher orders of a group. This result is very important for the order of every element of a group. This reason we can find out the order of elements of a group of different orders of a group. In different situations, once it was found that a given solution satisfies the basic result of one structure, and having known the properties of that structure, it becomes extremely easy to forecast the behavior of the situation. This result in this paper will be advantages for group theory related to subgroups and order of elements of a group. Thus, the demands of the work of mathematical problems as like as physical problems such as groups, number systems, vectors, matrices and so on.

REFERENCES

[1]. Marshall Hall Jr., David Wales, "The simple group of order 604, 800," Journal of Algebra, Volume 9, Issue 4, Pages 417-450,August 1968. https://doi.org/10.1016/0021-8693 (68)90014-8
[2]. Brauer, R., Tuan, H.F, "On simple groups of finite order, "I Bulletin of the American Mathematical Society, Volume 51, Issue 10, Pages 756-766, October 1945. DOI: 10.1090/S0002-9904-1945-08441-9
[3]. Md. Abdul Mannan, Halima Akter and Samiran Mondal, "Evaluate All Possible Subgroups of a Group of Order 30 and 42 By Using Sylow's Theorem, " International Journal of Scientific & Engineering Research, Volume 12, Issue 11, P 139-153, January-2021. ISSN 2229-5518,
[4]. Md. Abdul Mannan, Md. Amanat Ullah, Uttam Kumar Dey, Mohammad Alauddin , "A Study on Sylow Theorems for Finding out Possible Subgroups of a Group in Different Types of Order," Mathematics and Statistics, Vol. 10, No. 4, pp. 851 - 860, 2022. DOI: 10.13189/ms.2022.100416.
[5]. Mannan, M. A., Akter, H. ,& Ullah, . M. A. , " Evaluate All The Order of Every Element in The Higher Even, Odd, and Prime Order of Group for Composition, " Science and Technology Indonesia, 7(3), P 333–343, 2022. https://doi.org/10.26554/sti.2022.7.3.333-343
[6]. L. FINKELSTEIN AND A. RUDVALIS, "The Maximal Subgroups of Janko's Simple Group of Order 50, 232, 960, "JOURNAL OF ALGEBRA, Volume 30, P122-143,1974.
[7]. J. MCKAY AND D. WALES, "The multipliers of the simple groups of order 604, 800 and 50, 232, 960, " J. Algebra, 17, P 262-273, 1971.
[8]. Ulderico Dardano, Silvana Rinauro, "Groups with many Subgroups which are Commensurable with some Normal Subgroup, " Advances in Group Theory and Applications, 7, PP 3–13, 2019. DOI: 10.32037/agta-2019-002
[9]. S. Trefethen, "Non-Abelian composition factors finite groups with the CUT-property," J. Algebra, 522,P 236–242, 2019.
[10]. Mannan, Md.A., Nahar, N., Akter, H., Begum, M., Ullah, Md.A. and Mustari, S. "Evaluate All the Order of Every Element in the Higher Order of Group for Addition and Multiplication Composition, " International Journal of Modern Nonlinear Theory and Application, 11, P 11-30, 2022. https://doi.org/10.4236/ijmnta.2022.112002
[11]. D. J. S. Robinson, "A Course in the Theory of Groups," Springer-Verlag, New York, 1982.
[12]. Brendan McCann, "On Products of Cyclic and Non-Abelian Finite p-Groups," Advances in Group Theory and Applications, pp. 5–37,2020, ISSN: 2499-1287 DOI: 10.32037/agta-2020-001
[13]. D. Gorenstein, R. Lyons and R. Solomon, "The Classification of the Finite Simple Groups, Number 3," Mathematical Surveys and Monographs, Volume 40, Amer. Math. Soc., 1998.
[14]. M. Hall, "On the Number of Sylow Subgroups in a Finite Group," J. Algebra, 7, P 363–371, 1961.