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3D Mathematical Modeling and Simulation of Heart Movements in Normal and Atrial Fibrillation States

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Abstract: The heart's three-dimensional (3D) motion is a complex physiological process critical for efficient blood circulation. Abnormalities such as atrial fibrillation (AF) disrupt the synchronized contraction, leading to clinical complications. This paper presents a mathematical model for 3D heart motion under normal and fibrillated conditions, followed by simulation-based visualization using Python. The model is grounded in biomechanical principles and fluid-structure interactions, with numerical approximations to represent dynamic tissue deformation.

Keywords: *Atrial Fibrillation, Arrhytmia, Heart Motion, Electromechanical Coupling, Mechanical Deformation.*

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I. INTRODUCTION

Heart modeling aims to capture mechanical and electrical dynamics in both healthy and pathological states. Atrial fibrillation, the most common arrhythmia, affects millions globally [1]. The goal is to create 3D mathematical representations and simulations to compare normal and AF motions.

II. MATIHEMATICAL MODELING OF HEART MOTION

➤ *Basic Geometry of the Heart*

The heart is approximated as a prolate spheroid for simplicity [2, 3]. The deformation of the cardiac tissue is modeled using time-varying strain tensors:

$$\mathbf{X}(t) = \mathbf{R}(t) \cdot \mathbf{X}_0 + \mathbf{u}(t)$$

where $\mathbf{X}_0$ is the initial position, $\mathbf{R}(t)$ is the rotation matrix, and $\mathbf{u}(t)$ is the displacement vector.

➤ *Electromechanical Coupling*

The bidomain model governs electrical propagation:

$$\chi C_m \frac{\partial V_m}{\partial t} = \nabla \cdot (D_i \nabla V_i - D_e \nabla V_e) - I_{\text{ion}}$$

where V_m is the transmembrane potential, and D_i, D_e are the intracellular and extracellular conductivities [4, 5].

➤ *Mechanical Deformation*

Using nonlinear elasticity theory:

$$\rho \frac{d^2\mathbf{u}}{dt^2} = \nabla \cdot \mathbf{T} + \mathbf{f}$$

with $\mathbf{T}$ the Cauchy stress tensor and $\mathbf{f}$ the body force [6].

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III. NORMAL AND ATIAL FIBRILLATION

A. Normal Heartbeat

The **PQRST** cycle of a healthy ECG signal can be modeled as a sum of Gaussian functions:

$$ECG_{normal}(t) = \sum_{i=1}^{5} A_i \cdot e^{-\frac{(t-\mu_i)^2}{2\sigma_i^2}}$$

- A_i: amplitude of P, Q, R, S, T waves
- μ_i: center of each wave
- σ_i: width of each wave

B. Atrial Fibrillation (AF)

AF can be modeled as an irregular, noisy ECG-like signal with no consistent P wave and erratic R-R intervals:

$$ECG_{af}(t) = A \cdot \sin(2\pi f(t)t + \phi(t)) + \text{noise}$$

Where:

- $f(t)$: varying frequency
- $\phi(t)$: random phase
- noise: random normal variation

AF is introduced by randomizing activation sites and disturbing the periodic nature of excitation [7]. It leads to irregular mechanical strain and chaotic contractile behavior [8].

IV. NUMERICAL SIMUL ATION

The finite difference method is employed to simulate motion. The tissue contraction and relaxation are modeled as sinusoidal and randomized functions:

$$r(t) = r_0 + A \cdot \sin(\omega t) \quad \text{(Normal)}$$

$$r(t) = r_0 + A \cdot \sin(\omega t + \phi(t)) \quad \text{(AF)}$$

where $\phi(t)$ is a time-varying phase disturbance.

- *Python Program:*

3D Visualization of Normal and AF Heartbeats

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.animation import FuncAnimation

def create_heart_mesh():
theta = np.linspace(0, 2 * np.pi, 100)
z = np.linspace(-1, 1, 50)
theta, z = np.meshgrid(theta, z)
r = 1 - z**2  # heart shape radial deformation
x = r * np.cos(theta)
y = r * np.sin(theta)
return x, y, z

def update_heart_motion(x, y, z, t, af=False):
A = 0.1
omega = 2 * np.pi * 1.2
```

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```
phi = np.random.uniform(-np.pi, np.pi, size=x.shape) if af else 0
r_mod = 1 + A * np.sin(omega * t + phi)
return r_mod * x, r_mod * y, z

x0, y0, z0 = create_heart_mesh()

fig = plt.figure(figsize=(12, 6))
ax1 = fig.add_subplot(121, projection='3d')
ax2 = fig.add_subplot(122, projection='3d')
ax1.set_title('Normal Heart Motion')
ax2.set_title('Atrial Fibrillation Motion')

def animate(t):
ax1.cla()
ax2.cla()
xn, yn, zn = update_heart_motion(x0, y0, z0, t, af=False)
xa, ya, za = update_heart_motion(x0, y0, z0, t, af=True)
for ax, X, Y, Z in [(ax1, xn, yn, zn), (ax2, xa, ya, za)]:
ax.plot_surface(X, Y, Z, color='red', alpha=0.7, rstride=1, cstride=1)
ax.set_xlim([-1.5, 1.5])
ax.set_ylim([-1.5, 1.5])
ax.set_zlim([-1.5, 1.5])
ax.axis('off')

ani = FuncAnimation(fig, animate, frames=np.linspace(0, 2*np.pi, 50), interval=100)
plt.tight_layout()
plt.show()
```

V. DISCUSSION

The simulation reveals that normal contractions are periodic and symmetrical, while AF introduces spatial and temporal chaos.[See Figure-1 , Figure-2 and Figure-3] Such modeling can assist in diagnostic software, treatment planning, and biomedical device design [9–15].

VI. CONCLUSION

This paper presented a 3D mathematical framework and simulation for normal and fibrillated heart movement. Future work may involve real patient data integration and coupling with CFD for blood flow modeling.[16-30]

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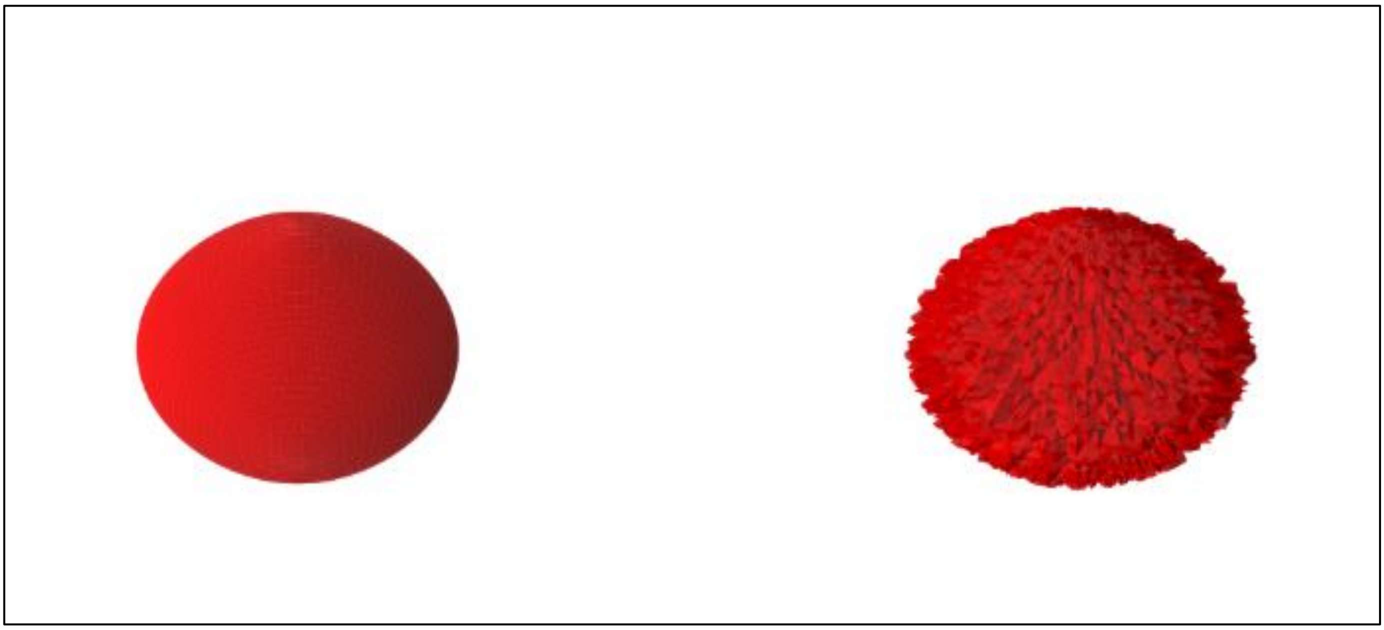
Fig 1 Normal Heart (Left) and Heart in Atrial Fibrillation(Right) in 3D+

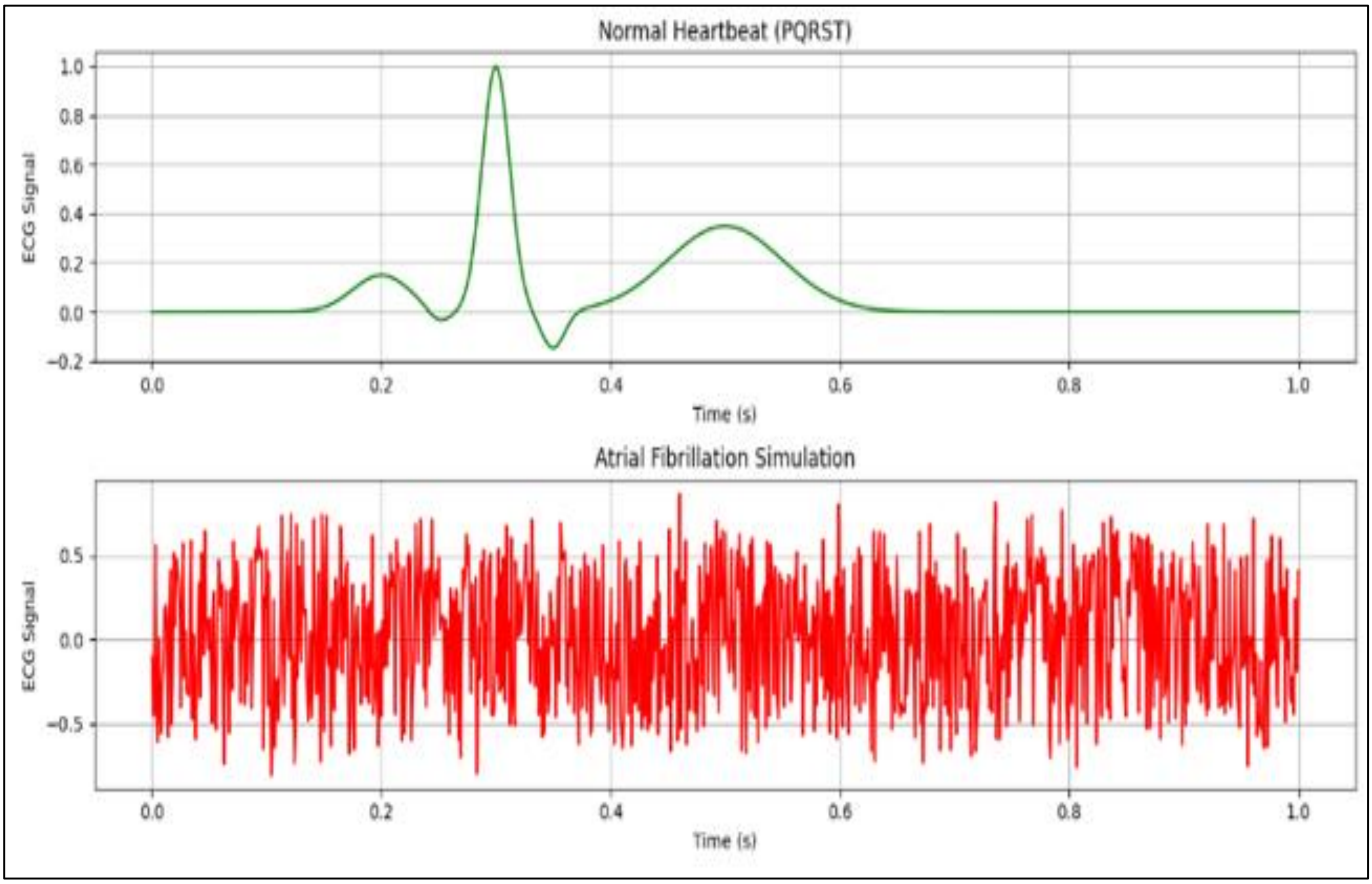

Fig 2 Normal and Attrial Fibrillation Heart in 2D

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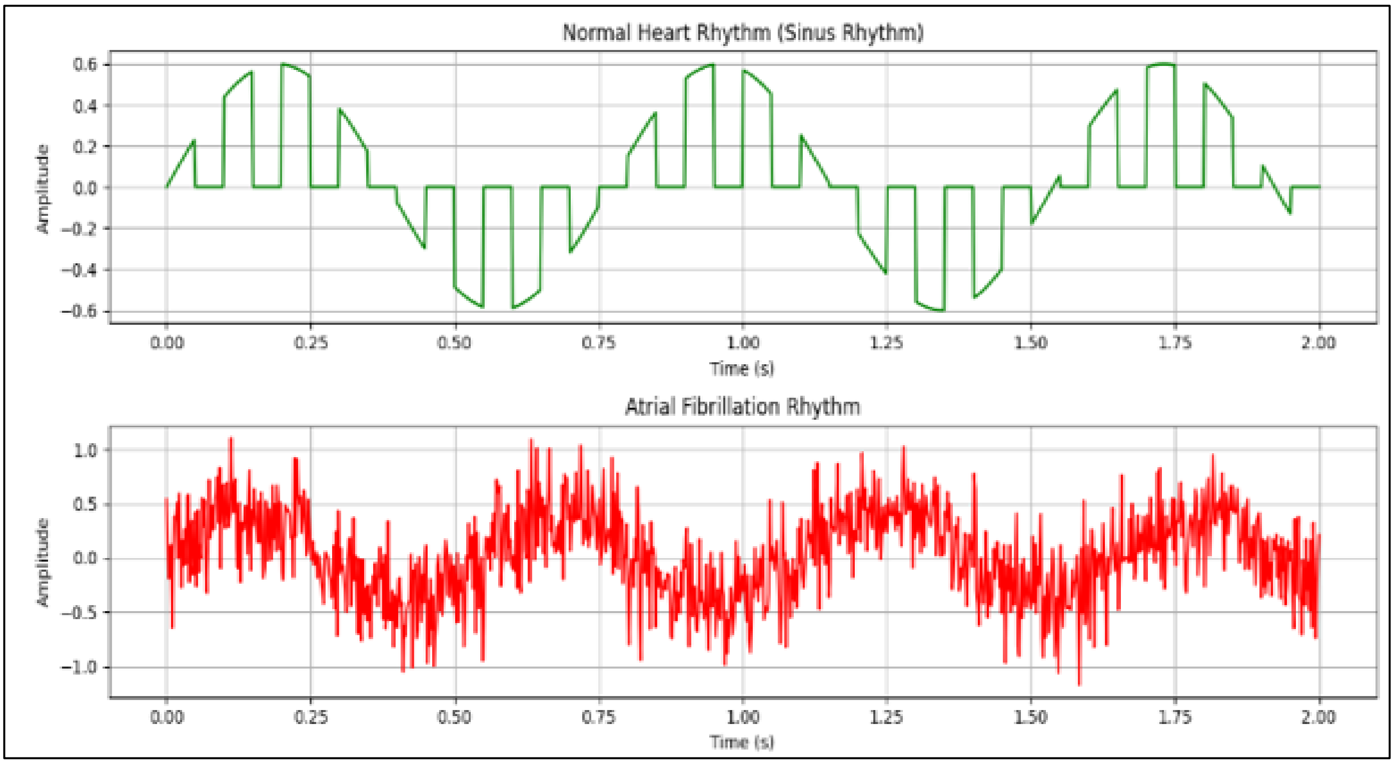

Fig 3 Normal Heart Beat and Attrial Fibrillation-Another Case

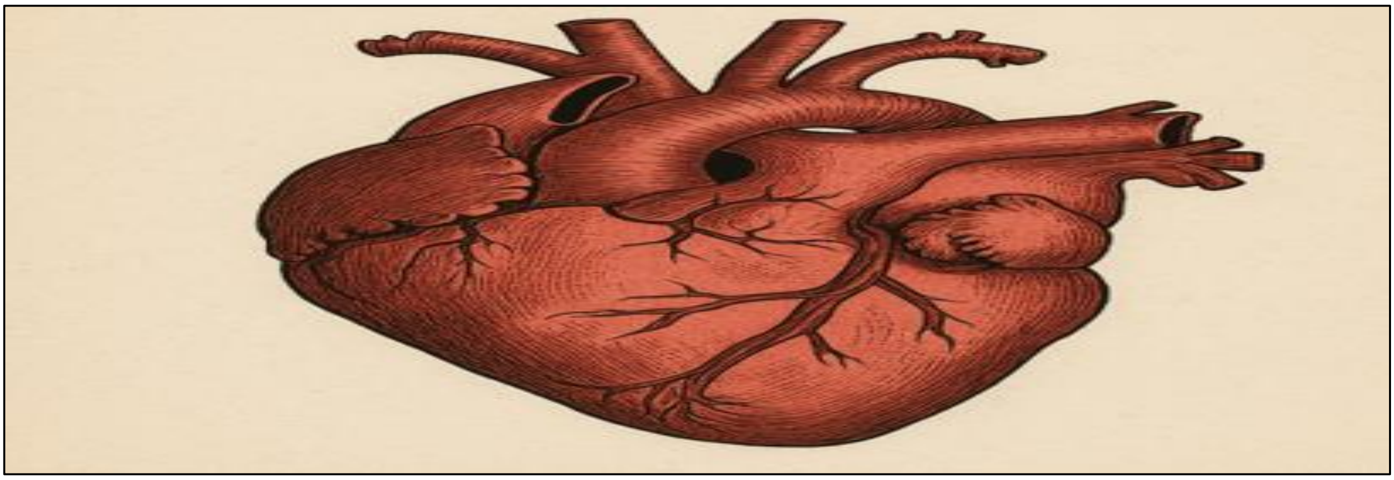

Fig 4 Normal Heart

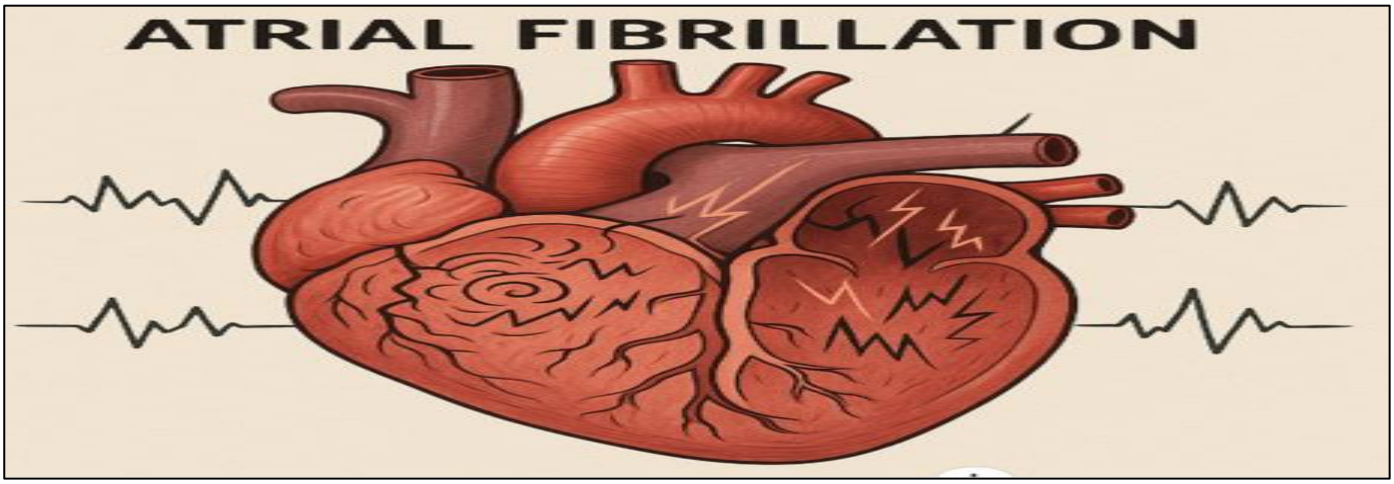

Fig 5 Heart in Atrial Fibrillation